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ANALYSIS OF FREQUENCY-CONTROLLED INDUCTION MOTOR AT HIGHER THAN RATED FREQUENCIES AND A CONSTANT STATOR VOLTAGE

A technique for calculation of characteristics of an induction motor operating under conditions: the constant load torque or constant load power at frequencies higher than rated ones and the rated stator voltage is presented. For the mode of constant load power the analytical solution of frequency-controlled induction motor electric drive transients is obtained.

1. INTRODUCTION

The article continues the previously published investigations of the author [1]–[3] with opening the new sides of induction motor operation at higher frequencies and rated stator voltage for a load with constant torque or constant power.

The induction motor velocities higher than rated ones are required for mechanisms operating under a constant load torque which is less than rated torque on some intervals of a timing load profile. For example, in the hoisting crane mechanisms on demand of high capacity it is profitable to hoist and to lower the light weights, say a hook, at velocities higher than nominal one.

The second example for mechanisms of a periodic duty may be the open-side or closed-side planer where the cutting of metal is performed only during the working stroke. The return stroke is idle. During the return stroke the force of resistance to motion is considerably less than force during working stroke. Therefore it is reasonable to use a higher velocity for the return stroke. But in some planers, existing till now, the higher return stroke velocity is reached via changing a train gear ratio. The expedient solution to this problem is to adjust the velocity of working and return

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strokes using a frequency-controlled induction motor with two ranges of frequency changing: 1) from zero to rated frequency and 2) from the rated to higher frequency. In doing so we come up against the problem of the allowable load torque and motor current for a maximum value of velocity. The present-day variable frequency converters (FC) at rated output frequency have a maximum value of the first harmonic output voltage with the most use of DC voltage. It can be seen from modern-day frequency converters data where space vector pulse width modulation (SV PWM) and three-phase diode bridge rectifiers are used under supply voltage of 400 V. Under these conditions the second range of frequency control induction motor angular velocity is carried out at a constant value of the first harmonic voltage output of FC, i.e. at $U_{\text{out}} = U_{\text{rat}} = \text{const}$ and frequency changing over the range $f_{1\text{rat}} \le f_1 \le f_{1\max}$.

In the second frequency range for induction motors of medium and high power it can be admitted that the active resistance of stator winding R_1 is zero compared with the reactances that are proportional to frequency. On this assumption the maximum electromagnetic torque M_m of a 3-phase induction motor varies in inverse proportion to the square of per unit frequency α [1]:

$$M_m = \frac{3U_{\text{rat}}^2}{2\omega_{0\text{rat}} x_{sh.\text{rat}} \alpha^2} = \frac{M_{m\text{rat}}}{\alpha^2} \tag{1}$$

whereas the absolute maximum slip is constant

$$S_{am} = \frac{R_2'}{x_{sh.\text{rat}}} = s_{m\text{rat}} \tag{2}$$

where

$$M_{m\text{rat}} = \frac{3U_{\text{rat}}^2}{2\omega_{0\text{rat}} x_{sh.\text{rat}}} \tag{3}$$

$$x_{sh.\text{rat}} = x_{1\text{rat}} + x_{2\text{rat}}' \tag{4}$$

$$\alpha = \frac{f_1}{f_{1\text{rat}}} \tag{5}$$

where:

$M_{m\text{rat}}$, $s_{m\text{rat}}$ – maximum electromagnetic torque and the absolute maximum slip of induction motor at $R_1 = 0$,

$f_{1\text{rat}}$ and U_{rat}, U_{rat} – rated phase stator voltage of induction motor,

$x_{1\text{rat}}$, $x_{2\text{rat}}'$ – leakage reactances of the stator and rotor (reduced to the stator) at rated frequency,

$x_{sh.\text{rat}}$ – the short-circuit reactance at rated frequency,

R_2' – phase resistance of rotor (reduced to the stator),

α – p.u. frequency,
$f_{1\text{rat}}$ – rated frequency of induction motor.

When $R_1 = 0$, we can obtain the following speed-torque curves of induction motor in p.u. representation [1]:

$$\left.\begin{aligned} \mu &= \frac{2\lambda_m}{\alpha^2\left(\dfrac{s_a}{s_{am}} + \dfrac{s_{am}}{s_a}\right)}, \\ \nu &= \alpha - s_a, \end{aligned}\right\} \tag{6}$$

where

$$\mu = \frac{M}{M_{\text{rat}}}, \tag{7}$$

$$\lambda_m = \frac{M_{m.\text{rat}}}{M_{\text{rat}}}, \tag{8}$$

$$s_a = \frac{\omega_0 - \omega}{\omega_{0\text{rat}}} = \frac{\Delta\omega}{\omega_{0\text{rat}}}, \tag{9}$$

$$\nu = \frac{\omega}{\omega_{0\text{rat}}}, \tag{10}$$

where:

M, ω – electromagnetic torque and angular velocity of induction motor,
M_{rat} – rated torque,
ω_0 – synchronous angular velocity of induction motor at given frequency f_1,
s_a – absolute slip of induction motor at given frequency f_1,
λ_m – maximum torque ratio under rated values,
μ, ν – p.u. torque and angular velocity for arbitrary conditions.

Supposing that the correction factor [4] is $\sigma_1 = 1 + \dfrac{x_{1\text{rat}}}{x_{\mu\text{rat}}} \approx 1$, one can derive the following relationships in p.u. values for:

- reduced rotor current

$$i_2' = \frac{I_2'}{I_{2\text{rat}}'} = \frac{\sqrt{\left(\dfrac{R_2'}{s_{\text{rat}}}\right)^2 + x_{sh.\text{rat}}^2}}{\beta\sqrt{\left(\dfrac{R_2'}{s_a}\right)^2 + x_{sh.\text{rat}}^2}} \tag{11}$$

- stator current

$$i_1 = \frac{I_1}{I_{1\text{rat}}} = \frac{Z_{IM}(s_{\text{rat}})}{Z_{IM}(s_a)}. \tag{12}$$

By introducing: I_1, I'_2 – r.m.s. currents of stator and rotor for arbitrary conditions, $I_{1\text{rat}}$, $I'_{2\text{rat}}$ – r.m.s. rated currents of stator and rotor, ZI_M, R_{IM}, X_{IM} – impedance, resistance and reactance for equivalent circuit of induction motor, $x_{\mu.\text{rat}}$ – magnetizing circuit reactance at rated frequency:

$$Z_{IM}(s_{\text{rat}}) = \sqrt{R_{IM}^2(s_{\text{rat}}) + X_{IM}^2(s_{\text{rat}})}, \tag{13}$$

$$R_{IM}(s_{\text{rat}}) = R_1 + R_{ia}(s_{\text{rat}}), \tag{14}$$

$$X_{IM}(s_{\text{rat}}) = x_{1\text{rat}} + x_{ia}(s_{\text{rat}}), \tag{15}$$

$$R_{ia}(s_{\text{rat}}) = \frac{x_{\mu.\text{rat}}^2 R'_2 s_{\text{rat}}}{R_2'^2 + (x'_{2\text{rat}} + x_{\mu.\text{rat}})^2 s_{\text{rat}}^2}, \tag{16}$$

$$x_{ia}(s_{\text{rat}}) = x_{\mu.\text{rat}} \frac{R_2'^2 + x'_{2\text{rat}}(x'_{2\text{rat}} + x_{\mu.\text{rat}}) s_{\text{rat}}^2}{R_2'^2 + (x'_{2\text{rat}} + x_{\mu.\text{rat}})^2 s_{\text{rat}}^2}, \tag{17}$$

$$Z_{IM}(s_a) = \alpha \sqrt{R_{ia}^2(s_a) + (x_{1\text{rat}} + x_{ia})^2}, \tag{18}$$

$$R_{ia}(s_a) = \frac{x_{\mu.\text{rat}}^2 R'_2 s_a}{R_2'^2 + (x'_{2\text{rat}} + x_{\mu.\text{rat}})^2 s_a^2}, \tag{19}$$

$$x_{ia}(s_a) = x_{\mu.\text{rat}} \frac{R_2'^2 + x'_{2\text{rat}}(x'_{2\text{rat}} + x_{\mu.\text{rat}}) s_a^2}{R_2'^2 + (x'_{2\text{rat}} + x_{\mu.\text{rat}})^2 s_a^2}, \tag{20}$$

Further we shall consider the characteristics and properties of induction motor in the second range of frequency-controlled angular velocity for: 1) constant load torque and 2) constant load power.

2. PROPERTIES AND CHARACTERISTICS OF INDUCTION MOTOR IN THE SECOND RANGE OF FREQUENCY CONTROL UNDER CONSTANT LOAD TORQUE

The expression (11) for the p.u. rotor current i'_2 can be converted as follows:

$$i_2' = \frac{s_a}{\alpha s_{\text{rat}}} = \frac{\sqrt{1+\left(\frac{s_{rat}}{s_{m.\text{rat}}}\right)^2}}{\beta\sqrt{1+\left(\frac{s_a}{s_{m.\text{rat}}}\right)^2}} \tag{21}$$

At the steady state of induction motor its electromagnetic torque M is equal to the load torque M_l, then in p.u. form we have $\mu = \mu_l$, where:

$$\mu_l = \frac{M_l}{M_{\text{rat}}}, \tag{22}$$

is the p.u. load torque.

Substituting (22) in (6) and rearranging terms one can obtain a quadratic equation for the absolute slip s_a. A solution to this equation can be found as:

$$s_a = \frac{s_{m.\text{rat}}}{\mu_l \alpha^2}(\lambda_m - \sqrt{\lambda_m^2 - \mu_l^2 \alpha^4})\,. \tag{23}$$

Now, knowing a given value of μ_l, it is possible to calculate s_a with (23) for anyone of α and then to find a solution for the p.u. rotor current i_2' by means of (21).

Under the previously taken assumption one can see that a p.u. maximum torque μ_m for any of a p.u. frequency α is expressed as

$$\mu_m = \frac{\lambda_m}{\alpha^2}\,. \tag{24}$$

In order to calculate a p.u. stator current i_1 with (12) a value $Z_{IM}(s_{\text{rat}})$, determined by (13)–(17), should be specified beforehand and thereafter a value $Z_{IM}(s_a)$ is ascertained through the use of formulae (18) – (20).

The p.u. dependences ν, i_1, i_2', μ_m as functions of α over the range $1 \le \alpha \le 2$ and $\mu_l = 0.386$ are presented in Fig. 1 for the induction motor of 4A160M6 type that has the rated data and parameters: $P_{\text{rat}} = 15$ kW; $U_{\text{rat}} = 380/220$ V; $s_{\text{rat}} = 0.03$; $\eta_{\text{rat}} = 87.5\%$; $\cos\varphi_{\text{rat}} = 0.87$; $x_{\mu.\text{rat}} = 22.107\ \Omega$; $x_{1\text{rat}} = 0.737\ \Omega$; $x_{2\text{rat}}' = 1.179\ \Omega$; $R_1 = 0.457\ \Omega$; $R_2' = 0.206\ \Omega$; $s_{m.\text{rat}} = 0.105$; $\lambda_m = 1.93$; $J_m = 0.18$ kgm^2; $f_{1\text{rat}} = 50$ Hz.

From Figure 1 one can see almost a linear increase of rotor i_2' and stator i_1 currents as the p.u. frequency α raises, at the same time the maximum torque μ_m is decreased in inverse proportion to the square of α.

When an induction motor operates over the range of p.u. frequencies $1 \le \alpha \le \alpha_{\max}$, it is necessary to have the induction motor maximum torque M_m at a maximum angular velocity $\omega_{\max}$ more than the load torque M_l by a required value, i.e.:

$$\frac{M_m(\omega_{\max})}{M_l(\omega_{\max})} = \lambda > 1\,. \tag{25}$$

For a frequency-controlled induction motor we can write the following acceptable relationships between p.u. values of frequencies and angular velocities

$$\alpha = \frac{f_1}{f_{1\text{rat}}} = \frac{\omega_0}{\omega_{0\text{rat}}} \approx \frac{\omega_{\max}}{\omega_{\text{rat}}} = D \tag{26}$$

where D is a range of the angular velocity control at $\alpha > 1$.

On the basis of (8), (24) and (26) the equality between the maximum torque M_m and the range of velocity changing D can be represented in a form:

$$M_m = \frac{M_{m.\text{rat}}}{D^2} = \frac{\lambda_m M_{\text{rat}}}{D^2}\,. \tag{27}$$

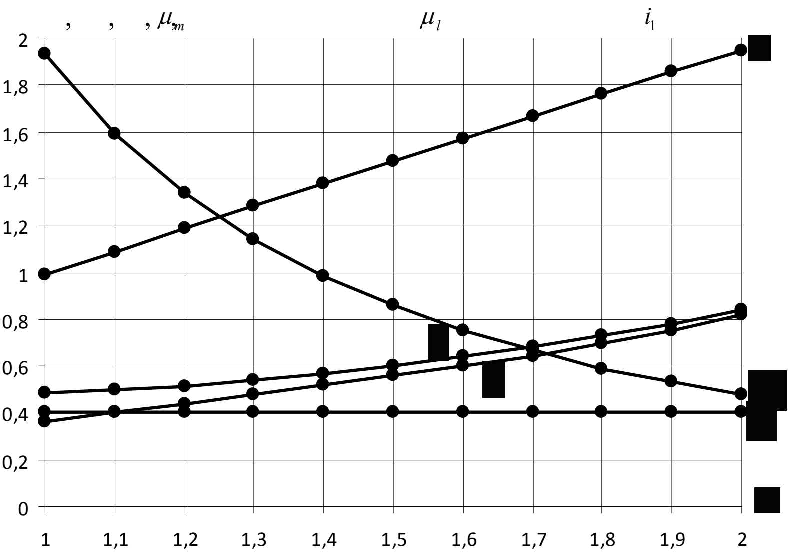

Fig. 1. Characteristics in p.u. of the induction motor 4A160M6, 15 kW, 380/220 V as a function of p.u. frequency α over the range $1 \le \alpha \le 2$ and under a p.u. load torque $\mu_l = 0.386$

From the relationships (25)–(27) the allowable load torque at a maximum angular velocity can be determined for a given power rating P_{rat} [3]:

$$M_l \le \frac{\lambda_m}{\lambda D^2} M_{\text{rat}} \tag{28}$$

where

$$M_{\text{rat}} = \frac{P_{\text{rat}}}{\omega_{\text{rat}}}, \tag{29}$$

M_{rat}, ω_{rat} are the rated torque and rated angular velocity of induction motor, λ is a permissible overload ratio at the angular velocity $\omega_{\max}$.

In p.u. the expression (28) transforms into one:

$$\mu_l \le \frac{\lambda_m}{D^2}. \tag{30}$$

In order to linearize the speed-current curve $\nu = F(i_2', \alpha)$ of induction motor we take into account that $s_a < s_{m.\text{rat}}$ and $s_{\text{rat}} < s_{m.\text{rat}}$.

This correlation allows us to write the approximate equality

$$\left(\frac{s_a}{s_{m.\text{rat}}}\right)^2 \approx \left(\frac{s_{\text{rat}}}{s_{m.\text{rat}}}\right)^2 \approx 0$$

in comparison with a value of 1 under the square root in (21).

As a result, the p.u. rotor current can be presented in a form:

$$i_2' = \frac{s_a}{\alpha s_{\text{rat}}}. \tag{31}$$

A linearized speed-torque curve $\nu = F(\mu, \alpha)$, where ν is shown through the α and s_a, is given in the book [1]:

$$\mu = \frac{2\lambda_m s_a}{\alpha^2 s_{m.\text{rat}}}. \tag{32}$$

Determing s_a from (32) and substituting it in (31), we obtain a formula for the p.u. rotor current i_2' in the type:

$$i_2' = \frac{\alpha \mu s_{m.\text{rat}}}{2\lambda_m s_{\text{rat}}}. \tag{33}$$

The formula (33) shows that under a condition $\mu = \mu_l = \text{const}$ the p.u. rotor current increases linearly when α rises. In actually, the rotor current increase will be a little steeper than a linear dependence that may be seen from Fig. 1 where the p.u. rotor current i_2' was calculated by (21).

3. PROPERTIES AND CHARACTERISTICS OF INDUCTION MOTOR IN THE SECOND RANGE OF FREQUENCY CONTROL UNDER CONTANT LOAD POWER

A series of electrical drives for industrial applications such as the electrical drives for main motion of machine tools and traction electrical drives require a mode of operation with the constant load power for velocities higher than rated ones. In the past years the mode of load power constancy was obtained by means of weakening the magnetic field in a separate excited DC motor (SE DCM) under the rated armature voltage. The SE DCM and induction motors have the same dependence of weakening the magnetic field, but their electromagnetic torques vary differently when the magnetic flux changes. When weakening the magnetic flux in the SE DCM, its electromagnetic torque varies as the reciprocal of the angular velocity, whereas the induction motor electromagnetic torque changes in inverse proportion to the square of the angular velocity.

If the induction motor operates over the range of p.u. frequencies $1 \leq \alpha \leq \alpha_{\max}$, it is necessary to select power rating in such a manner in order to ensure at $\alpha_{\max}$ the required overload ratio qualified by technological conditions. Under the constancy of load power P_l the load torque at the maximum angular velocity will be equal to

$$M_l(\omega_{\max}) = \frac{P_l}{\omega_{\max}} \tag{34}$$

that gives a value of the overload ratio

$$\lambda = \frac{\lambda_m M_{\text{rat}} \omega_{\max}}{P_l \alpha_{\max}^2} \tag{35}$$

Let us write (35) as

$$\lambda \alpha_{\max}^2 P_l = \lambda_m M_{\text{rat}} \omega_{\max} \tag{36}$$

Multiplying the left and right side of (36) by rated angular velocity ω_{rat}, we get an equality:

$$\lambda \alpha_{\max}^2 P_l \omega_{\max} = \lambda_m P_{\text{rat}} \omega_{\max} \tag{37}$$

where P_{rat} is a power rating of induction motor.

Based on (37) and using (26), we can derive a formula to calculate the needed power rating of induction motor assuming the values P_l, D, λ_m and λ are known

$$P_{\text{rat}} = \frac{\lambda}{\lambda_m} D P_l . \tag{38}$$

When all over the range D of frequency control angular velocity it is necessary to hold a constant overload ratio equaled to $\lambda = \lambda_m$, the power rating, as you may see from (38), will be D times higher than the load power P_l. Such application is ineffective variant of using the frequency-controlled induction motor at the rated voltage and higher frequencies. But for some mechanisms, for instance, a grinding machine, at a maximum velocity it is allowed to take $\lambda < \lambda_m$. Then, the induction motor power rating will differ from the load power by the acceptable value [3].

For a mode of load power constancy the p.u. load torque can be presented in such a manner:

$$\mu_l = \frac{K_l}{\alpha} \tag{39}$$

where

$$K_l = \frac{M_{l.\text{rat}}}{M_{\text{rat}}} = \frac{P_{l.\text{rat}}}{P_{\text{rat}}}, \tag{40}$$

$M_{l.\text{rat}}$, $P_{l.\text{rat}}$ are the rated load torque and rated load power.

In the connection that $K_l \le 1$ the formula (38) can be used for determining the allowable load power P_l if magnitudes P_{rat}, D, λ_m, λ are specified.

Substitution of (39) in (32) permits us to come close to a constant value:

$$i_2' = \frac{K_l s_{m.\text{rat}}}{2\lambda_m s_{\text{rat}}} = \text{const} . \tag{41}$$

Really, the p.u. rotor current i_2' with an increase α is not constant but reduces in some degree that can be seen from Fig. 2.

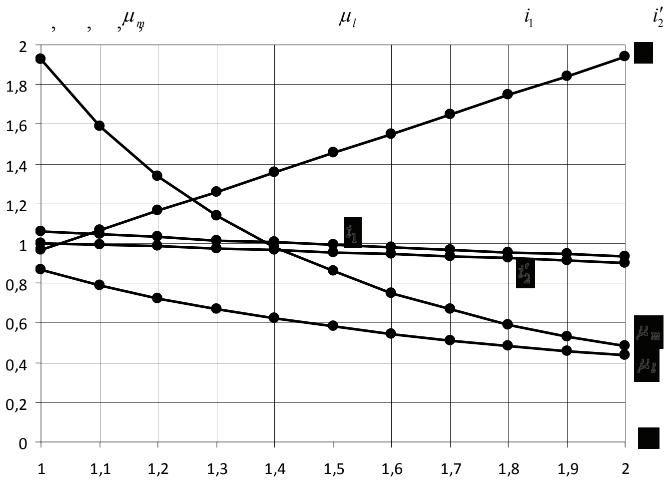

Fig. 2. Characteristics in p.u. of the induction motor 4A160M6, 15 kW, 380/220 V as a function of p.u. frequency α over the range $1 \le \alpha \le 2$ and under a p.u. load power $K_l = 0.87$

Hence, if the absolute slip s_a changes by law

$$s_a = \alpha s_{\text{rat}}, \tag{42}$$

the rotor current, as one can see from (32), will have a constant magnitude but the electromagnetic torque determined by (32) will be changed as the reciprocal of the p.u. frequency α

$$\mu = \frac{2\lambda_m s_{\text{rat}}}{\alpha s_{m.\text{rat}}}. \tag{43}$$

So, if the absolute slip varies as mentioned above, the frequency-controlled induction motor carries out a mode of load power constancy. The p.u. stator current i_1 is determined by (12) where $Z_{IM}(s_a)$ is calculated through (18) but "inner" resistances $R_{ia}(s_a)$ and reactances $x_{ia}(s_a)$ are specified via (19)–(20) taking into account (42), i.e.:

$$R_{ia}(s_a) = \frac{\alpha x_{\mu.\text{rat}}^2 R_2' s_{\text{rat}}}{R_2'^2 + \alpha^2 (x_{2\text{rat}}' + x_{\mu.\text{rat}})^2 s_{\text{rat}}^2} \tag{44}$$

$$x_{ia}(s_a) = x_{\mu.\text{rat}} \frac{R_2'^2 + \alpha x_{2\text{rat}}' (x_{2\text{rat}}' + x_{\mu.\text{rat}})^2 s_{\text{rat}}^2}{R_2'^2 + \alpha^2 (x_{2\text{rat}}' + x_{\mu.\text{rat}})^2 s_{\text{rat}}^2} \tag{45}$$

As an illustration, the characteristics in p.u. of the induction motor 4A160M6, P_{rat} = 15 kW, U_{rat} = 380/220V are presented in Fig. 2 for the frequency range $1 \le \alpha \le 2$ and P_l = 13 kW = const when the absolute slip s_a changes in accordance with (42).

The p.u. rotor current presented by (21) when the condition (42) is considered can be rearranged as follows:

$$i_2' = \frac{\sqrt{1 + \left(\dfrac{s_{\text{rat}}}{s_{m.\text{rat}}}\right)^2}}{\sqrt{1 + \alpha^2 \left(\dfrac{s_a}{s_{m.\text{rat}}}\right)^2}}. \tag{46}$$

From Figure 2 it can be seen that at the maximum velocity corresponding to $\alpha_{\max}$ = 2 the overload torque ratio λ is about 1.1, and the stator and rotor currents descend a little when α increases that is in compliance with the theory.

4. MECHANICAL TRANSIENTS OF FREQUENCY-CONTROLLED INDUCTION MOTOR ELECTRICAL DRIVE UNDER THE CONSTANT LOAD POWER

When the absolute slip s_a changes by law (42), one can obtain the analytical solution for mechanical transients of induction motor electrical drive. If variables in the equation of motion

$$M - M_l = j\frac{d\omega}{dt} \tag{47}$$

to express in p.u. terms of torque and angular velocity and to use a linearised speed-torque curve, one can derive the differential equation

$$\mu - \mu_l = T_M \frac{d\nu}{dt} \tag{48}$$

where

$$\nu = \alpha(1 - s_{\text{rat}}), \tag{49}$$

$$\mu = \frac{2\lambda_m s_{\text{rat}}}{\alpha s_{m.\text{rat}}}, \tag{50}$$

$$\mu_l = \frac{K_l}{\alpha}, \tag{51}$$

$$T_M = \frac{J\omega_{0.\text{rat}}}{M_{\text{rat}}}, \tag{52}$$

$$J = K_J J_m, \tag{53}$$

where:

- J – total moment of inertia reduced to the motor shaft,
- J_m – moment of inertia of induction motor,
- K_J – factor of inertia,
- T_M – the time constant equalled to the electrical drive starting time under the zero load torque, rated moving torque and rated frequency.

Substituting (49)–(51) in (48) we obtain the differential equation for a p.u. frequency α:

$$\alpha\frac{d\alpha}{dt} = \frac{1}{(1 - s_{\text{rat}})T_M}\left(\frac{2\lambda_m s_{\text{rat}}}{s_{m.\text{rat}}} - K_l\right). \tag{54}$$

Taking into account that

$$\alpha \frac{d\alpha}{dt} = \frac{1}{2}\frac{d(\alpha^2)}{dt} \tag{55}$$

and denoting

$$A = \left(\frac{2\lambda_m s_{\text{rat}}}{s_{m.\text{rat}}} - K_l \right) \frac{2}{(1 - s_{\text{rat}})T_M}\,, \tag{56}$$

$$x = \alpha^2\,, \tag{57}$$

we find the differential equation in such a form:

$$\frac{dx}{dt} = A \tag{58}$$

a solution of which is

$$x = \alpha^2 = At + C \tag{59}$$

where C is a constant of integration specified from initial conditions, it equals α_{in}^2 .

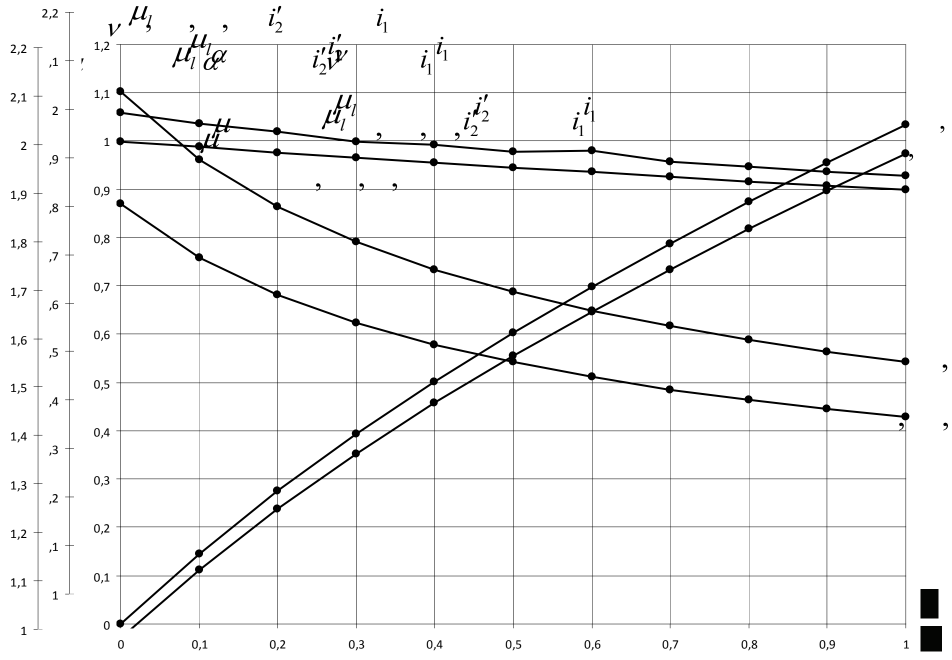

Fig. 3. Dependencies in p.u. of mechanical transients for an electrical drive on the basis of induction motor 4A160M6, 15 kW, 380/220 V during acceleration and maintaining the load power constancy

As a consequence, we ascertain that during electrical drive transients a p.u. frequency α varies in time t in accordance with the dependency

$$\alpha = \sqrt{At + \alpha_{in}^2} \ . \tag{60}$$

The rest variables of transients: ν, μ, μ_l, i_2', i_1 are functions of α and can be calculated by corresponding formulae, i.e., (49)–(51), (46), (12) together with (13)–(15) and (44), (45).

The p.u. variables α, μ, μ_l, i_2', i_1 of induction motor electrical drive mechanical transients are shown in Fig. 3 for the induction motor 4A160M6, P_{rat} = 15 kW, U_{rat} = 380/220 V under conditions P_l = 13 kW (K_l = 0.87), K_J = 1.2 and changing α from 1 to 2.

5. TENDENCY IN DEVELOPMENT OF FREQUENCY-CONTROLLED INDUCTION MOTOR ELECTRIC DRIVES FOR A MODE OF CONSTANT LOAD POWER

The main drawback in creation of a mode P_l = const by means of variation the absolute slip via (42) is that an overload torque ratio of induction motor decreases as the p.u. frequency α grows. When $\alpha_{\max} \approx \lambda_m$ the overload torque ratio approaches to 1 that is intolerable for many applications. The constant overload torque ratio of induction motor may be achieved at the cost of an increase the power rating as it follows from (38), but in most cases such a solution is economically unacceptable, especially, for $\alpha_{\max} > 2$ and high magnitudes of load power.

A rational approach to realization of load power constancy with an invariable overload torque ratio contains in M.P. Kostenko's law for frequency control of induction motors [4]:

$$\gamma = \frac{U}{U_{\text{rat}}} = \sqrt{\alpha} \ . \tag{61}$$

But at once a question arises: where can one take voltage more than the rated at p.u. frequency $\alpha > 1$? Some foreign companies such as Siemens and Baldor perform the two-range frequency control of induction motor in such a manner: over the first frequency range $0 < \alpha \leq 1$ an induction motor operates with the stator winding connection Y but through the second frequency range $1 < \alpha \leq \alpha_{\max}$ the stator windings are changed over from Y to Δ connection. At the same time a depth of SV PWM drops from a maximum value in the end of the first range to the value that ensures the fulfillment of a voltage changing law (61). It is obviously that under such switching of stator windings we get equality

$$\gamma_{\max} = \sqrt{\alpha_{\max}} = \sqrt{3}\,.$$

Hence, in such an electrical drive the velocity control range of $D = 3$ is achieved at a constant value of overload torque ratio through all the range.

If $\alpha_{\max} > 3$ is required, one can attain a range 4–5, of course, under the lesser overload torque ratio of induction motor. The catalogue data of foreign companies demonstrate the possibilities to do so.

Two-speed induction motors with switching stator windings from Y to Δ and using a variable frequency converter in each range of frequency control are mentioned by Siemens, for example, the electrical drive Simodrive 611.

6. CONCLUSIONS

1. The dependencies of stator and rotor currents increasing in the second frequency range for load power constancy have been determined, and the analytical expression for calculation of the maximum load torque at given velocity range and power rating of induction motor has been obtained.
2. For the load power constancy over a given range of velocity change a formula to calculate the power rating of induction motor when the load power is known and vice versa to calculate the load power at given power rating of induction motor has been derived.
3. Using the linearization of induction motor speed-torque and speed-current curves in the second frequency range, the analytical solution to electrical drive mechanical transients has been discovered.

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