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MATLAB-SIMULINK MODELLING OF INDUCTION MACHINE INCORPORATING MAGNETIC SATURATION

1. INTRODUCTION

The subject of this paper is the mathematical model of the cage induction machine incorporating effect of magnetic saturation suitable for Matlab-Simulink programming environment. As the base for modelling the previously published network model of the machine was used [1, 3, 4] and introduced to the computer program in form of Simulink S-function. Connection of this S-function with models of electrical and mechanical environment is assured by controlled sources. In the computer program the stator winding can be delta or star connected with and without the ground neutral. Hence, for these connections the zero components of currents or voltages caused by saturation can be measured in the model. Using the SimPower Systems library the machine model can cooperate with models of various static converters, e.g. inverters, cycloconverters and AC regulators working as a motor or an induction generator. Additionally the control techniques developed for the machine model with linear magnetic circuit (e.g. the flux oriented control) can be applied to check the influence of saturation.

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2. MATHEMATICAL DESCRIPTION OF THE INDUCTION MACHINE

The general form of voltage equations of an induction machine can be formulated for every winding and can be written using the matrix description (subscript 1 – before transformation to symmetrical components):

$$\begin{bmatrix} \mathbf{U}_{s1} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{s1} & \\ & \mathbf{R}_{r1} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{s1} \\ \mathbf{I}_{r1} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \mathbf{\Psi}_{s1} \\ \mathbf{\Psi}_{r1} \end{bmatrix} \tag{1}$$

In the above: $\mathbf{U}_{s1} = [u_U \; u_V \; u_W]^T$, $\mathbf{I}_{s1} = [i_U \; i_V \; i_W]^T$ – vectors of stator voltages and currents respectively, $\mathbf{I}_{r1} = [i_1 \; i_2 \; i_3 \ldots i_N]^T$ – vector of rotor cage mesh currents, $\mathbf{\Psi}_{s1} = [\psi_U \; \psi_V \; \psi_W]^T$ – vector of stator phase flux linkages, $\mathbf{\Psi}_{r1} = [\psi_1 \; \psi_2 \; \psi_3 \ldots \psi_N]^T$ – vector of rotor cage mesh flux linkages, $\mathbf{R}_{s1}$ – matrix of stator phase resistances, $\mathbf{R}_{r1}$ – matrix of rotor cage elements resistances.

Saturation of the magnetic core is caused in the greatest degree by fundamental harmonic of total MMF [5]. This part of MMF producing the pole pair number p takes the following form versus the angle x calculated along the air gap of the machine cross section

$$F_1(x) = F_{1s}(x) + F_{1r}(x) = F_m \cos p(x - \alpha_m) \tag{2}$$

The MMF magnitude can be substituted with the magnitude i_m of equivalent magnetising current according to $F_m = 2\sqrt{3}\frac{N_w k_w}{\pi\, p} i_m$, where: N_w – number of stator winding turns, k_w – stator winding factor for fundamental harmonic. The first maximum of MMF fundamental harmonic is in the place α_m. Hence, the permeance appears as the response of exciting MMF described by (2). The field crosses the air gap in $2p$ places. So, the permeance function takes the simplest description incorporating effect of magnetic saturation and can be described by the Fourier series [3, 4, 6]:

$$\Lambda = \Lambda_o \sum_{\mu=0,2} \Lambda_\mu^{pu}(i_m) \cos[\mu p(x - \alpha_m)]\,; \qquad \Lambda_o = \frac{4\mu_o R l_c}{\pi p^2 \delta} \tag{3}$$

In the above: $\mu_o = 4\pi \cdot 10^{-7}$ H/m, R – internal radius of the stator core, l_c – equivalent length of the machine core, δ – air-gap equivalent length. Modelling the non-linearity of the magnetic core using the permeance function changes seemingly the air gap of the machine from smooth to salient.

The MMF harmonics must satisfy the following relationship for their numbers ν and ρ

$$|\nu \pm \rho| = \mu \in \{0, 2\} \tag{4}$$

These harmonic numbers belong to one set H determining accuracy of MMF approximation. The model presented in this paper incorporates the lowest possible set H = {1, 3}. The coefficient $Q = 1$ indicates that equation (4) is satisfied. If not, then $Q = 0$. Hence, $Q = 1$ for the following threes of ν, ρ and μ: (1, 1, 0), (1, 1, 2), (1, 3, 2), (3, 1, 2), (3, 3, 0).

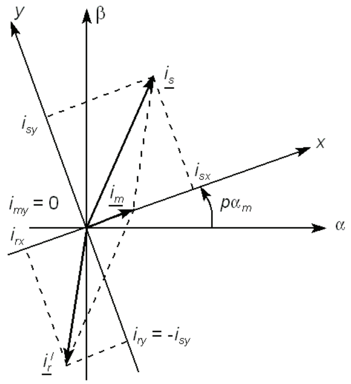

Fig. 1. Illustration for transformation from the stationary reference frame *α–β* to the magnetising current oriented *x–y*

Transforming the set of equations (1) to the symmetrical components in the stationary reference frame *α–β* and next to the reference frame *x–y* determined by the position $\alpha = p\alpha_m$ of the magnetising current vector $\underline{i_m} = i_{m\alpha} + ji_{m\beta} = i_m e^{j\alpha}$ the set of equations assumes the following form [3, 4] (the transformation is explained by Fig. 1):

$$\mathbf{U_2} = \mathbf{R_2}\mathbf{X_2} + \mathbf{M_2}\frac{d}{dt}\mathbf{X_2} + \mathbf{E_2} \tag{5}$$

In this set the stator voltages and the state variables exist in vectors:

$$\mathbf{U_2} = [\tfrac{1}{2}u_s^{(0)}, u_{sx}, u_{sy}, 0, 0]^T, \quad \mathbf{X_2} = [i_s^{(0)}, i_{sx}, i_{sy}, i_{rx}, \alpha] \tag{6}$$

The magnetising current becomes the modulus

$$i_m = i_{sx} + i_{rx} \tag{7}$$

$u_s^{(0)} = \frac{1}{\sqrt{3}}(u_U + u_V + u_W)$ – zero component of stator voltages, u_{sx}, u_{sy} – components of the stator voltage vector in the *x–y* reference frame, $i_s^{(0)} = \frac{1}{\sqrt{3}}(i_U + i_V + i_W)$ – zero component of stator currents, i_{sx}, i_{sy} – components of the stator current vector in the *x–y* reference frame, i_{rx} – *x* component of the rotor current vector. Parameters of (5) can be taken from [3, 4].

Electromagnetic torque is given by general expression as a partial derivative of magnetic co-energy versus rotor rotation angle. From this expression the dominant part was taken giving the simplified formula.

3. MATLAB–SIMULINK MODELLING

For computer modelling the set of differential equations describing the machine was introduced to S-function coded in Matlab language. Connection between the model and the external circuit (Fig. 2) was made by transformation from the rotating *x–y* frame to the 3-phase system using controlled current sources representing stator phase windings of the machine (Fig. 3). These current sources are controlled with the phase currents i_U, i_V, i_W calculated after solving the set of differential equations and inverse Park's transformation from model currents $i_s^{(0)}$, i_{sx}, i_{sy}. They exit the S-function becoming the control signals. The voltage drops on these currents sources are the phase voltages u_U, u_V, u_W that are transmitted back to the machine model in the S-Function block.

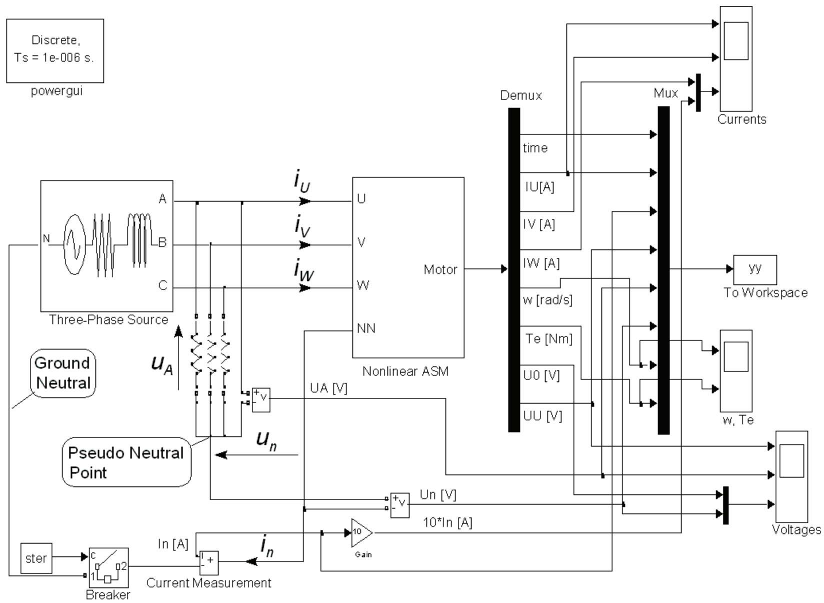

Fig. 2. The main system connecting the saturating induction machine with the external circuit (here: Three-Phase Source) in Simulink

To avoid an algebraic loop the discrete delay elements were introduced into the model. Additionally the current sources must be shunted with resistors of relatively large resistance, e. g. 100 kΩ. Such a measure protects the computer program against numerical checking of the Kirchhoff's current law. This speeds up computations.

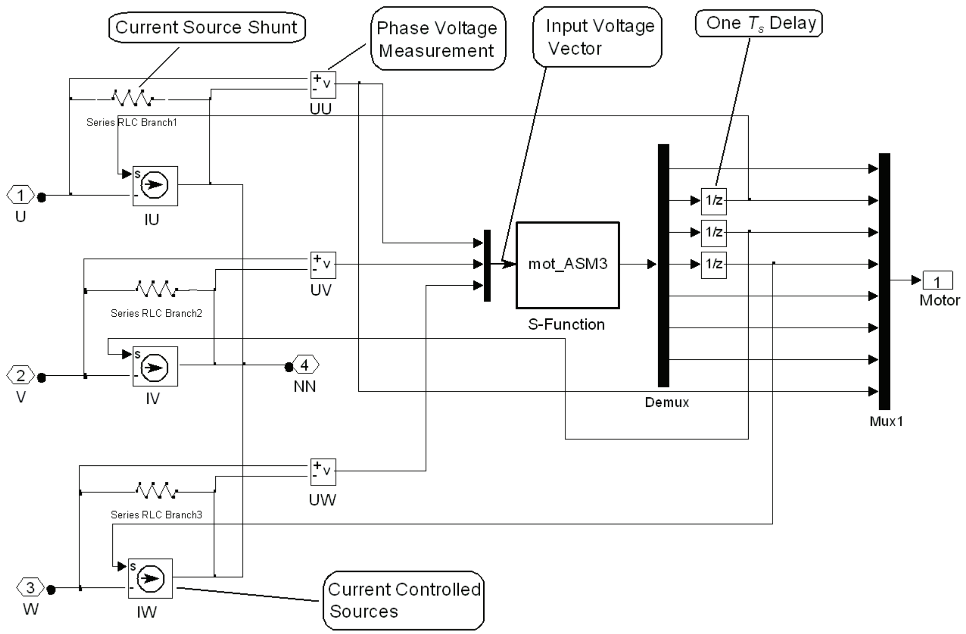

Fig. 3. Simulink model of a non-linear induction machine with the set of differential equations **mot_ASM3** establishing S-Function

4. RESULTS OF SIMULATION

To illustrate operation of the proposed modelling method the simulations of typical states for the induction machine working as a motor were performed. These were: free acceleration starting to the steady state at the star connection with the ground neutral (Fig. 4) and without this neutral connection (Fig. 5). The presented waveforms were taken from terminals of the machine as in the real system. The mathematical model was formulated for the induction motor Sf112M4 (P_N = 4 kW, $U_{N(ph\text{-}ph)}$ = 660 V Y, f_{sN} = 50 Hz, p = 2).

The zero symmetrical current component $i_s^{(0)} = \frac{i_n}{\sqrt{3}}$ and the phase current i_U have typical shapes for loaded and unloaded motors – appropriate measurements were presented in [4]. The change of the phase current shape is caused by its shift in phase with respect to the supplying voltages.

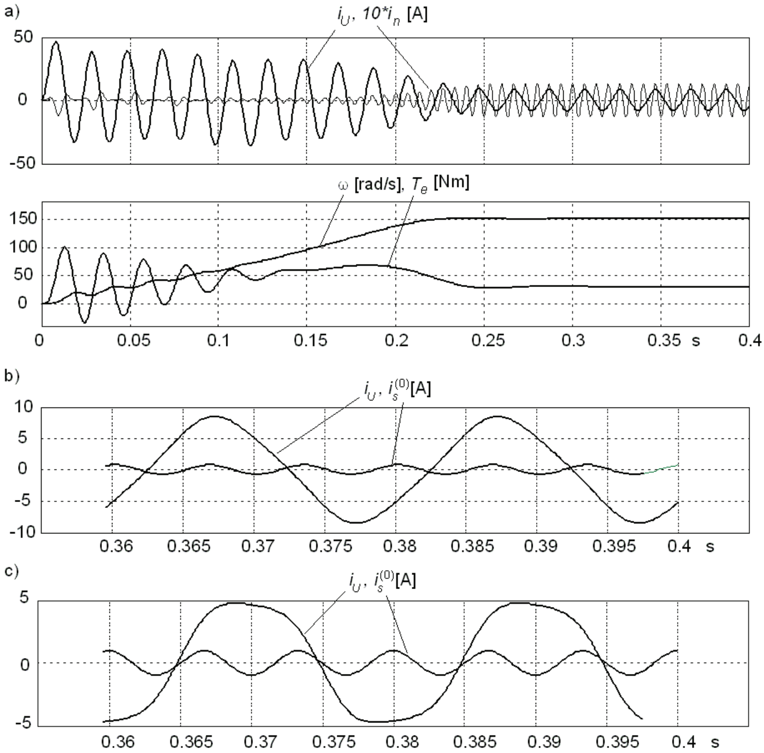

Fig. 4. Free acceleration starting of loaded induction motor (a) (connection: in star with the ground neutral) and steady state waveforms of phase current and the stator current zero symmetrical component: b) for loaded motor, c) for unloaded motor

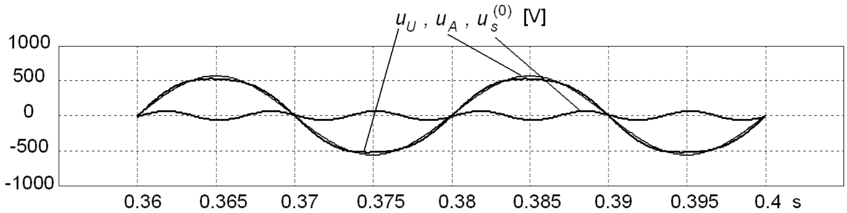

Fig. 5. Steady state waveforms of phase voltages (for the motor and the mains) and the zero voltage symmetrical component

The work without the ground neutral gives typical shape of the measured neutral voltage u_n with respect to the supplying phase voltage u_A and the voltage u_U induced in the phase winding. This voltage is distorted additionally by the machine slotting [4]. The zero symmetrical component of voltages descends from u_n according to the relationship $u_s^{(0)} = \sqrt{3}\, u_n$.

5. CONCLUSIONS

The presented model is the simplest one comprising the phenomenon of magnetic saturation in the induction machine and gives characteristic signals of current and voltage zero symmetrical components. This model was implemented into Matlab--Simulink programming as the S-Function and connected with current controlled sources to the external circuit (3-phase voltage source, inverter). The current sources representing the phase windings can be star and delta connected. Thus, this approach extends Simulink spectrum of application in spite of the library model of saturated induction motor offered by the last version (2011) of Matlab programming – there is the model accessed only as a three wire box.

The presented method for using the S-Function can be applied to the mathematical models of the other electrical machines, models of sophisticated electromechanical converters and other devices (not especially electrical). This is obvious that the model can be used for induction generator modelling.

Additionally, sensitivity of MRAS methodology [7], based on the linear model, can be trained using this model incorporating magnetic core non-linearity.

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